Two Means

Unpaired / Independent

Parameter: $\mu_1 - \mu_2$
Point est.: $\bar{x}_1 - \bar{x}_2$

Conditions for Dist^n Approximation

1. Indep. observations in both samples
2. Nearly normal pop^ns or Large ($n > 30$) Sample Sizes
3. Independently selected samples

Hypothesis Test

Hypotheses

$H_0: \mu_1 - \mu_2 = 0$

H_a: ① $\mu_1 - \mu_2 < 0$
② $\mu_1 - \mu_2 \neq 0$
③ $\mu_1 - \mu_2 > 0$

Test Statistic RV (Assuming H_0)

$$T = \frac{(\bar{X}_1 - \bar{X}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \sim t_{\min(n_1 - 1, n_2 - 2)}$$

Observed Test Stat

$$t_{obs} = \frac{(\bar{x}_{1,obs} - \bar{x}_{2,obs}) - 0}{\sqrt{\frac{s_{1,obs}^2}{n_1} + \frac{s_{2,obs}^2}{n_2}}}$$

P-value

① $\mathbb{P}(\bar{X}_1 - \bar{X}_2 \leq \bar{x}_{1,obs} - \bar{x}_{2,obs}) = \mathbb{P}(T \leq t_{obs})$
② $\mathbb{P}(|\bar{X}_1 - \bar{X}_2| \geq |\bar{x}_{1,obs} - \bar{x}_{2,obs}|) = \mathbb{P}(|T| \geq |t_{obs}|)$
③ $\mathbb{P}(\bar{X}_1 - \bar{X}_2 \geq \bar{x}_{1,obs} - \bar{x}_{2,obs}) = \mathbb{P}(T \geq t_{obs})$

Confidence Interval

Formula for CI

$$(\bar{x}_{1,obs} - \bar{x}_{2,obs}) \pm t^*_{df} \sqrt{\frac{s_{1,obs}^2}{n_1} + \frac{s_{2,obs}^2}{n_2}}$$

Paired

Parameter: μ_{diff}
Point est.: $\bar{x}_{diff}$

Conditions for Dist^n Approximation

1. Independent obs^ns among pairs
2. Nearly normal pop^n of differences OR Large # of pairs

Hypothesis Test

Hypotheses

$H_0: \mu_{diff} = 0$

H_a: ① $\mu_{diff} < 0$
② $\mu_{diff} \neq 0$
③ $\mu_{diff} \neq 0$

Test Statistic

$$T = \frac{\bar{X}_{diff} - 0}{s_{diff} / \sqrt{n}} \sim t_{n-1}$$

Obs. Test. Stat

$$t_{obs} = \frac{\bar{x}_{diff,obs} - 0}{s_{diff,obs} / \sqrt{n}}$$

P-value

① $\mathbb{P}(\bar{X}_{diff} \leq \bar{x}_{diff,obs}) = \mathbb{P}(T \leq t_{obs})$
② $\mathbb{P}(|\bar{X}_{diff}| \geq |\bar{x}_{diff,obs}|) = \mathbb{P}(|T| \geq |t_{obs}|)$
③ $\mathbb{P}(\bar{X}_{diff} \geq \bar{x}_{diff,obs}) = \mathbb{P}(T \geq t_{obs})$

Confidence Interval

Formula for CI

$$\bar{x}_{diff,obs} \pm \left[t^*_{df} \times \frac{s_{diff,obs}}{\sqrt{n}}\right]$$